## St Aldhelm's VA Church Primary School



## Calculation Policy Autumn 2018



## $2+1=3$ <br> ABSTRACT


(100) (00) $10 \quad 10 \quad 10$ (1)

This policy has been designed to teach children to develop conceptual understanding through the progression of concrete, pictorial and abstract methods. This calculation policy should be used to support children to develop a deep understanding of number and calculation.

## Using the concrete-pictorial-abstract approach:

Children develop an understanding of a mathematical concept through the three steps (or representation) of concrete-pictorial-abstract approach. Reinforcement is achieved by going back and forth between these representations.

Concrete representation The enactive stage - a pupil is first introduced to an idea or a skill by acting it out with real objects. This is a 'hands on' component using real objects and it is the foundation for conceptual understanding.

Pictorial representation The iconic stage - a pupil has sufficiently understood the hands-on experiences performed and can now relate them to representations, such as a diagram or picture of the problem.

Abstract representation The symbolic stage - a pupil is now capable of representing problems by using mathematical notation, for example: $12 \div 2=6$.

## Part/Whole Model - Key Structures:

Addition and Subtraction are connected. Add parts together to equal the whole, whole subtract part to name the missing part.


Pictorial Representation:
To aid consistency between year groups, the following images should be used and drawn by children to represent numbers. The children do not need to record the colours, simply the shapes, but where possible the colours should be used in teacher modelling and classroom resources.

| Million | Hundred <br> Thousand | Ten <br> Thousand | Thousand | Hundred | Ten | One |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\square$ |  |  |  |  |  |  |

## Early Years

## Developing Number Sense

## Vocabulary

Part, whole, add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on. equal to, take, take away, less, minus, subtract, leaves, difference between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_?

Ordinality:

Concrete:
Children place a range of physical dominoes in a set order.


## Pictorial:

Children match representations in a set order, for
example, using pictorial bear / number
dominoes.


## Abstract:

Children fill in spaces on a partially filled number track and create representations to show different totals (extension) - helping pupils to make the transition from understanding ordinality to cardinality.

|  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 |  | 3 | 4 |  | 6 | 7 | 8 |  | 10 |

## Ordinal numbers:

## Concrete:

Children physically line up ducks in a row and
verbally label them, e.g. 'first/second / third.'


## Pictorial:

Children order slides with pictures of ducks, for example, on the Interactive Whiteboard.


## Abstract:

Children apply their understanding of ordinal numbers, e.g. by using written $1^{\text {st }}, 2^{\text {nd }}$ and $3^{\text {rd }}$ labels and other related verbal language when ordering objects.


## Cardinality:

## Concrete:

Children use a range of structured and unstructured apparatus, plus natural resources, to create different number values.


## Pictorial:

Children recognise different number values that are presented in pictorial forms.


## Abstract: <br> Children are asked a range of questions that

 allow them to show an application of understanding related to cardinality, e.g.Can you find a collection of...[objects]...to
represent six?
Can you show me six fingers?

## Subitising:

| Concrete: |
| :--- | :--- | :--- |$\quad$| Pictorial: |
| :--- |
| Children replicate a range of physical |
| representations, which they then verbally |
| interpret without a need to count objects. |



## 1 to 1 correspondence:

| Concrete: | Pictorial: <br> Children count various physical objects by <br> dice. <br> partitioning a group and finally recombining. | Abstract: <br> Children draw dots to match the number of holes <br> that can be seen on a named Numicon shape. |
| :--- | :--- | :--- |
| Children write a number in each part of a muffin <br> tin and then put the appropriate number of a pictorial <br> buttons in each section. | Children match number cards to pictures of the <br> equal numbers of buttons. | Children cut out buttons equal to the number <br> shown on a number card. |

## Conservation of number:

Concrete:
Children explore whether the number of cubes stay the same or change when they are moved within a shape.


Pupils also count dolls and then put them in different rooms before re-counting to check the total. Hopefully they decide that if nobody has left and nobody has arrived, then it must be the same total even if some of the dolls have moved rooms.


Pictorial:
Pupils work with visual reminders of their concrete experiences - to check how their understanding around conservation of number has changed.


## Abstract:

Children are provided with opportunities to further explore and prove their thinking. They may be asked to put a total of dolls in the toy house and then move them around. In order to prove it is still the same total, they can take the dolls and put them onto a number track, whilst also applying their understanding about the cardinal principle.


## Concept of zero:

## Concrete:

Children use a shuffle box with up to ten objects in . After the box has been shaken, pupils write out the corresponding number sentence, e.g. $2=$ $1+1$, depending on where the objects have landed. Query what happens if there is nothing on one side. Introduce to children the concept of zero, e.g. $2=2+0$.


Pictorial:
Children use pictorial representations to see that you can have an amount that's called 'zero.' Pupils are required to count the number of apples of a tree, and circle the trees which have no apples.


Abstract:
Children can be encouraged to represent written number sentences by creating visual shuffle boxes using finger paint, e.g. $5=0+5$


Pupils should be able to grasp the concept of zero to use within number sentences, e.g. $4=4+0 \ldots$ and verbalise ...
"I know that four is the same as four add zero."

## Counting on:

Concrete:
Children use physical objects to learn the skill. For example, they count on from the larger value by using their fingers whilst pointing at each 'extra' dot on the second side of a domino.


In addition, pupils use counters on number tracks to rehearse the process of counting on.


## Pictorial:

Children use a die to generate numbers and count on from pictorial representations of counters already positioned on a number track.

Abstract:
Children apply their understanding of this skill by playing games such as 'snakes and ladders.'


## Reception

## Reception Addition

## Vocabulary

Part, whole, add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on. Use of Numicon is another great way to help children develop mental representations of number.


- Say which number is one more or one less than a given number.

These experiences and number representations will help children

- Reliably count the number of objects in a set using the numbers one to twenty.




## Year 1

## Year 1 Addition

## Vocabulary

Part, whole, addition, add, forwards, put together, more than, total, altogether, distance between, difference between, equals = same as, most, pattern, odd, even, digit, counting on.


Learn number bonds to 20 and demonstrate related facts
Teach addition and subtraction alongside each other as pupils need to see the relationship between the facts.

|  | $\begin{aligned} & 6+4=10 \\ & 4+6=10 \\ & 10-4=6 \\ & 10-6=4 \end{aligned}$ | $\text { (10) } \begin{aligned} & 6+4=10 \\ & 4+6=10 \\ & 10-4=6 \\ & 10-6=4 \end{aligned}$ | 10 |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  | 6 | 4 |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
| Tens Frame |  | Part Whole Model | Bar Model |  |



| Regrouping to make 10 ; using ten frames and counters/cubes or using Numicon. $\begin{aligned} & 6+5 \\ & 0 \end{aligned}$ <br>  | Children to draw the ten frame and counters/cubes. | Children to develop an understanding of equality e.g. $\begin{aligned} & 6+\square=11 \\ & 6+5=5+\square \\ & 6+5=\square+4 \end{aligned}$ |
| :---: | :---: | :---: |

## Year 1 Subtraction

## Vocabulary

Part, whole, subtraction, subtract, take away, distance between, difference between, more than, minus, less than, equals = same as, most, least, pattern, odd, even, digit,

$$
\begin{array}{l|l}
\text { Physically taking away and removing objects from a whole } & \begin{array}{l}
\text { Children to draw the concrete resources they are using } \\
\text { and cross out the correct amount. The bar model can }
\end{array} \\
\text { (ten frames, Numicon, cubes and other items such as }
\end{array}
$$

beanbags could be used).

## $4-3=1$


also be used.

$4-3=$


## Subtraction by counting back

Counting back (using number lines or number tracks) children start with 6 and count back 2.
$6-2=4$


Children to represent what they see pictorially e.g.


Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line


Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used).

Calculate the difference between 8 and 5 .


Making 10 using ten frames.
14-5


When subtracting using Dienes children should be taught to regroup a ten rod for 10 ones and then subtract from those ones

$20-4=16$

## Subtracting multiples of 10

Using the vocabulary of 1 ten, 2 tens etc alongside 10,20,30 Is very important here as pupils need to understand that it is a 10 not a 1 that is being taken away


## Year 1 Multiplication

## Vocabulary

Part, whole, ones, groups, lots of, doubling, repeated addition, groups of, lots of, times, columns, rows, longer, bigger, higher etc and times as (big, long, wide ...etc)

## Counting in multiples of 2,5 and 10 from zero

Children should count the number of groups on their fingers as they are skip counting.
4 groups of $2=8$

$2 \times 4=8$
2
two
2
WO
2
two
2
two
When moving to pictorial/written calculations the vocabulary is important


This image represents two groups of 4 or 4 twice

## Year 1 Division

## Vocabulary

Part, whole, share, share equally, one each, two each..., group, groups of, lots of, array
Pupils should be taught to divide through working practically and the sharing should be shown below the whole to familiarize children with the concept of the whole.

The language of whole and part part should be used.
$8 \div 4=2$


There are 4 boxes of 2 cans.

## Year 2

## Year 2 Addition

Vocabulary
Part, whole, +, add, addition, more, plus, make, sum, total, altogether, how many more to make...? how many more is... than...? how much more is...? =, equals, sign, is the same as, tens, ones, partition Near multiple of 10 , tens boundary, more than, one more, two more... ten more.
Using concrete objects and pictorial representations to add 3 single digit numbers.


$$
7+3+2=\text { leads to } 10+2=
$$

Using concrete objects and pictorial representations to add a 2 digit number and ones and tens.
$25+3=28$
田


As with all written calculations, children must always start with the right hand column (ones). Encourage children to count on rather than just counting in ones i.e count on from 5 in this example. The box diagram can be compared to a window (4 boxes) and a windowsill (bottom row)

Using concrete objects and pictorial representations to add two 2-digit numbers

$$
15+34=49
$$




## Leading to:

In order to promote the mental maths skills needed for the abstract stage, when a regroup is needed, children should be encouraged to partition one of the numbers to make a bond to ten with the other i.e in this example partition 5 into 4 and 1 as the 4 makes the bond with the 6. This partitioning should be encouraged rather than simply counting in ones.

$$
36+25=61
$$

Looking for ways to make 10 .


1

10 ones are regrouped (exchanged) into one ten which can be drawn pictorially as a rod under the windowsill.

## Using the bar to find missing digits.

It is important for children to use the bar in this way to encourage the use of it to aid with problem solving.

Helen has 14 breadsticks. Her friend has 17. How many do they have altogether?


## Year 2 Subtraction

## Vocabulary

Part, whole, Subtraction, subtract, take away, difference, difference between, minus
Tens, ones, partition
Near multiple of 10, tens boundary, Less than, one less, two less... ten less.
Using concrete objects and pictorial representations to subtract a 1 -digit number from 2-digit number.


Using dienes and pictorial representations to subtract a 2-digit number from 2 digit number.

$$
68-32=36
$$



## Greater Depth:

A double cross should be used to show where a regroup (exchange) has been made so that it is not confused with the crossing out of subtraction.
Once regrouped, the subtraction should start from within the regrouped ten as this promotes the partitioning skills needed for the abstract stage.

$$
72-47=25
$$

丑


Recognise and use the inverse relationship between addition and subtraction

| 76 |  |
| :---: | :---: |
| 23 | $?$ |


| $?$ |  |
| :---: | :---: |
| 23 | 53 |

Use to check inverse calculations

## Year 2 Multiplication

## Vocabulary

Part, whole, multiple, multiplication array, multiplication tables / facts, groups of, lots of, times, columns, rows

Skip counting in multiples of 2, 3, 5, 10 from 0


Recall and use multiplication facts for the multiplication tables 2,5 and 10.

| Repeated grouping/repeated addition $3 \times 4$ $4+4+4$ <br> There are 3 equal groups, with 4 in each group. <br> Number lines to show repeated groups$3 \times 4$ <br> Cuisenaire rods can be used too. | Children to represent the practical resources in a picture and use a bar model. <br> Represent this pictorially alongside a number line e.g.: | $\begin{aligned} & 3 \times 4=12 \\ & 4+4+4=12 \end{aligned}$ <br> Abstractnumberline showingthree jumps of four. $3 \times 4=12$ |
| :---: | :---: | :---: |
| Use arrays to illustrate commutativity counters and other objects can also be used. $2 \times 5=5 \times 2$ <br> 2 lots of 5 <br> 5 lots of 2 | Children to represent the arrays pictorially. $\begin{array}{ll} 00 & 00000 \\ 00 & 00000 \\ 00 & \\ 00 & \\ 00 & \end{array}$ | Children to beable to use an array to writea range of calculations e.g. $\begin{aligned} & 10=2 \times 5 \\ & 5 \times 2=10 \\ & 2+2+2+2+2=10 \\ & 10=5+5 \end{aligned}$ |
| Vocabulary <br> Part, whole, group in pairs, 3s ... 10s etc | Year 2 Division <br> , equal groups of, divide, $\div$, divided | by, divided into, remainder |
| Sharing using a range of objects. $6 \div 2$ <br> Repeated subtraction using Cuisenaire rods above a ruler. $6 \div 2$ <br> 3 groups of 2 | Represent the sharing pictorially. <br> Children to represent repeated subtraction pictorially. | $6 \div 2=3$3 3 <br> Children should also be encouraged to use their 2timestablesfacts. <br> Abstract number line to represent the equal groups that have been subtracted. |



## Year 3

## Year 3 Addition

## Vocabulary

Part, whole, hundreds, tens, ones, estimate, partition, recombine, difference, decrease, near multiple of 10 and 100, inverse, rounding, column subtraction, exchange See also Y1 and Y2

## Add two three-digit numbers.

Children need to use equipment first to support their understanding of place value.
Children to progress gradually to three digit + three digit starting without carrying and gradually moving towards carrying.
As with all written calculations, children should start from the right hand side (ones in this example.

$243+368=611$

$243+368=611$
囲


243
$\begin{array}{r}+368 \\ \hline \frac{611}{11}\end{array}$

10 ones are regrouped (exchanged) into one ten 'under the windowsill'. Then 10 tens are regrouped (exchanged) into one hundred 'under the windowsill'.

In order to promote the mental maths skills needed for the abstract stage, when regrouping is needed, children should be encouraged to partition one of the numbers to make a bond to ten with the other i.e in this example partition 8 into 7 and 1 as the 7 is the bond with the 3 . This partition should be encouraged rather than simply counting in ones.

## Using the bar to find missing digits.

It is important for children to use the bar in this way to encourage the use of it to aid with problem solving.

Bar Model to support understanding of problem solving:
A man sold 230 balloons at a carnival in the morning. He sold another 86 balloons in the evening. How many balloons did he sell in all?


## Year 3 Subtraction

## Vocabulary

Part, whole, hundreds, tens, ones, estimate, partition, recombine, difference, decrease, near multiple of 10 and 100 , inverse, rounding, column subtraction, exchange see also Y 1 and Y 2

## Subtract up to 3 digits from 3 digits.

Very important for children to use dienes equipment along with a place value chart to support.

$$
263-121=142
$$



Only when secure with the method should exchanging be introduced.

$$
234-88=
$$

234-88=


234-88=


A double cross should be used to show where a regroup (exchange) has been made so that it is not confused with the crossing out of subtraction.

Once regrouped, the subtraction should start from within the regrouped ten as this promotes the partitioning skills needed for the abstract stage.

## Using the bar to find missing digits.

It is important for children to use the bar in this way to encourage the use of it to aid with problem solving.

| 315 |  | $315-185=?$ |
| :--- | :--- | :--- |
| 185 | $?$ | $185+?=315$ |


| $?$ |  | $185+315=?$ <br>  <br> 185 315 |
| :--- | :--- | :--- |

## Year 3 Multiplication

Vocabulary
Part, whole, multiple, partition, short multiplication and inverse
Children should be able to recall the $2,5,10,3,4$ and 8 times tables.
Multiply a two-digit number by a one digit.


Children use diennes and represent
them pictorially
$23 \times 6=$


Children may use the ladder method before the compact 23
$\times \quad 6$
$183 \times 6$
120 20x6
138

Formal witten method
$6 \times 23=$
23


138
11

Using the bar to solve multiplication problems.

4 children go to the cinema.
They each pay $£ 15$. How much do they spend altogether?

Whole unknown

| $?$ |  |  |  |
| :--- | :--- | :--- | :--- |
| 15 | 15 | 15 | 15 |

## Year 3 Division

## Vocabulary

Part, whole, See Y1 and Y2 and Inverse, remainder
Dividing using short division.


Remind children of correct place value, that 69 is equal to 60 and 9, but in short division, pose:
$\cdot$ How many 3 s in 6 ? $=2$, and record it above the 6 tens.
$\cdot$ How many 3 s in $9 ?=3$, and record it above the 9 ones.
Once children demonstrate a full understanding of remainders, and also the short division method taught, they can be taught how to use the method when remainders occur within the calculation (e.g. $42 \div 3$ ), and be taught to 'carry' the remainder onto the next digit.


As with addition and subtrcation, a double cross should be used to show a regroup i.e in this example one ten is regrouped into ten ones.

Using the bar to aid the solving of division problems - grouping and sharing


## Year 4

## Year 4 Addition

## Vocabulary

Part, whole, add, addition, sum, more, plus, increase, sum, total, altogether, double, near double, how many more to make..? how much more? ones boundary, tens boundary, hundreds boundary, thousands boundary, tenths boundary, hundredths boundary, inverse, how many more/fewer? Equals sign, is the same as.

## Adding numbers with up to 4 digits.

Again this should start with the children using dienes to support them with lots of discussion about the value of each digit.


## Using the bar to find missing digits.

It is important for children to use the bar in this way to encourage the use of it to aid with problem solving.
This is not a form of getting the correct answer but helping to guide children to the correct operation.

Alison jogs 6,860 metres and Calvin jogs 5,470 metres. How far do they jog altogether?

| $?$ |  |
| :---: | :---: |
| 6860 m | 5470 m |

## Year 4 Subtraction

## Vocabulary

Part, whole, subtract, takeaway, less, minus, decrease, fewer, difference, how many less to make..? how much less? ones boundary, tens boundary, hundreds boundary, thousands boundary, tenths boundary, hundredths boundary, inverse, how many fewer? Equals sign, is the same as.
To subtract with numbers up to four digits including exchanging when children are secure.
Children need to use dienes to support their learning.


Using the bar to find missing digits.
It is important for children to use the bar in this way to encourage the use of it to aid with problem solving.

There are 3,160 books in a shop. 1,226 are in English and the rest are in French. Howmany French books are there?


## Year 4 Multiplication

## Vocabulary

Part, whole, Factor, product

## Children to know all times tables to $12 \times 12$.

Children multiplying both two and three digits by a one digit number using dienes.
When using dienes, encourage children to use their tables knowledge rather than counting in ones i.e $6 \times 5$ rather than counting 30 ones.


## Multiplying using the bar.

A computer costs 5 times as much as a television. The television costs $£ 429$.

How much does the computer cost?

Cost of the computer


## Year 4 Division

## Vocabulary

Part, whole, see years 1-3, divide, divided by, divisible by, divided into, share between, groups of factor, factor pair, multiple, times as (big, long, wide ...etc), equals, remainder, quotient, divisor and inverse
Dividing up to three digit numbers by a one digit number using short division.
Only when the children are secure with dividing a two digit number should they move onto a 3 digit number.


With remainders


Remind children of correct place value, that 126 is equal to 100,20 and 6 , but in short division, pose:

- How many 5s in 12? = 2, and record it above the 2 tens.
- How many 5 s in 6 ? $=1$, and record it above the 1 one.

As with addition and subtraction, use a double cross for regroup.
Dividing using the bar.
Desmond and Melissa collect cards. They have 192 cards in all Melissa has three imes as many cards as Desmond. How many cards does Desmond have?


## Year 5

## Year 5 Addition

## Vocabulary

Part, whole, tens of thousands boundary,
Also see previous years

## Adding numbers with more than 4 digits including decimals

Using place value charts are key to this as well as place value counters to help with the decimals.


Using the bar to find missing digits.
It is important for children to use the bar in this way to encourage the use of it to aid with problem solving. This is not a form of getting the correct answer but helping to guide children to the correct operation.

MacDonald sold $£ 9957.68$ worth of hamburgers and $£ 1238.5$ worth of chicken nuggets. How much money did they take altogether?

|  | $?$ |
| :--- | :--- |
| $£ 957.68$ |  |
|  | $£ 1238.5$ |

## Year 5 Subtraction

## Vocabulary

Part, whole, tens of thousands boundary,
Also see previous years

## Subtract with at least four digit numbers including two decimal places.

Include money, measures and decimals ensuring that children do this practically before the abstract.
Subtract with decimal values, including mixtures of integers and decimals, aligning the decimal point.


## Using the bar to find missing digits.

It is important for children to use the bar in this way to encourage the use of it to aid with problem solving.
A whole to Lapland costs $£ 5005$ for a family of fo
the Smith's have only saved $£ 3787.75$, how much
money do they still need to find?

| $£ 5005$ |  |
| :---: | :---: |
| ? | $£ 3787.75$ |

## Year 5 Multiplication

## Vocabulary

Part, whole, cube numbers, prime numbers, square numbers, common factors, prime number, prime factors and composite numbers
Multiplying up to four digit numbers by two digits using long multiplication.
Children need to be taught to approximate first, egg. for $\mathbf{7 2 \times 3 8}$, they will use rounding: $\mathbf{7 2 \times 3 8}$ is
approximately $70 \times 40=\mathbf{2 8 0 0}$, and use the approximation to check the reasonableness of their answer.



## Using the bar to support multiplication.

The cost to run a sports centre is $£ 4375$ a
week, how much would it cost to run for 16 weeks?


## Year 5 Division

## Vocabulary see year 4

Part, whole, common factors, prime number, prime factors, composite numbers, short division, square number, cube number, inverse, power of
Diving with up to four digit numbers by one digit including numbers where remainders are left.

Short division with remainders:
examples that give rise to have a real life problem solving meaning of the remainder and decimal, or as a rounded context of the problem.


Now that pupils are introduced to remainder answers, division needs to context, where pupils consider the how to express it, ie. as a fraction, a number or value, depending upon the

Using the bar to support division problems.
Bar Model to support understanding of problem solving:
Frank has 4920 apples. He needs to put them into baskets of 40 . How many baskets does he need?

## Year 6 (supporting transition into Year 7)




## Year 6 Division

## Vocabulary

see years 4 and 5 Part, whole, long division
Divide at least 4 digits by both single -digit and 2-digit numbers (including decimal numbers and quantities)
Short division with remainders: Pupils should continue to use this method, but with
 numbers to at least 4 digits, and understand how to express remainders as fractions, decimals, whole number remainders, or rounded numbers. Real life problem solving contexts need to be the starting point, where pupils have to consider the most appropriate way to express the remainder.

Long division this is for when dividing by two digit numbers.


## Division

$564 \div 13$
13

$$
\begin{array}{r}
43.38 \\
\hline 56^{4} 4.0^{511} 0
\end{array}
$$

$564 \div 13$

$=43$ r $5=43 \frac{5}{13}=43.4$ (to 1 dp )

| Try this equation; $848 \div 16$ |
| :--- |
| Approximation $800 \div 16=50$ |



Using the bar to help divide.

> Paul and David hire a car together at a cost of
> $£ 297.50$. Paul pays 6 times more than David.

How much does David pay?


